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Glimm's Method Applied to Underwater Explosions

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The Random Choice Method, conceived by Glimm, and developed by Chorin, for solving hyperbolic partial differential equations, is applied to calculate the disturbance produced by a spherical underwater explosion. Results are compared with earlier calculations, under similar conditions, using the Method of Characteristics. Good qualitative agreement with earlier results is observed.

1. INTRODUCTION

The aim of this paper is to apply a recently developed technique to the calculation of the flow field generated by an explosion released at a small depth beneath the ocean surface. This problem has been investigated previously (see [1, 2]) using the classical Method of Characteristics and Shock Fitting.

The present numerical method used is a Random Choice Method proposed by Glimm [3] and developed by Chorin [4, 5]. This method is used for obtaining the numerical solutions of systems of nonlinear hyperbolic systems, particularly those arising in gas dynamics. Chorin has also developed the present method for hydrodynamics.

The main advantage of this method is its ability to track discontinuities, thus keeping shocks, slip lines, sharply defined, as opposed to other numerical methods which tend to smear out shocks and other discontinuities. This method also does not require the introduction of artificial viscosity. The present objective is to apply Chorin's Random Choice Method to the problem of underwater explosions, and observe how well it defines the shock and slip line boundaries. The shock is followed until it makes contact with the ocean surface, up to which time the flow field behind the shock is spherically symmetric.

The one-dimensional equations of motion for an inviscid, non-heat-conducting, radially symmetric flow can be written in the form

$$U_t + F(U)_r = -W(U), \tag{1}$$

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where

$$U=\begin{pmatrix}\rho\\ m\\ E\end{pmatrix},\qquad F(U)=\begin{pmatrix}m\\ m^2/\rho+p\\ m(E+p)/\rho\end{pmatrix},$$
$$W(U)=(\alpha-1)\begin{pmatrix}m/r\\ m^2/\rho r\\ m(E+p)/\rho r\end{pmatrix}. \tag{2}$$

In Eqs. (1) and (2) the independent variables are t, the time, and r, the radial distance from the center of the explosion. The unknowns are ρ, the density, u, radial velocity, p, pressure, E, specific total energy. The momentum flux ρu is denoted by m. Finally $\alpha=2$ in cylindrical flow and $\alpha=3$ in spherical flow. Subscripts indicate differentiation. For perfect gases we may write

$$E=p/(\gamma-1)+\tfrac{1}{2}\rho u^2, \tag{3}$$

where γ is the ratio of specific heats (a constant greater than 1). There are two problems in solving system (1) directly. First, there is a singularity at $r=0$. Second, the momentum equation cannot be written in conservation form. Sod [6] and Li and Holt [7] both used a judicious combination of operator splitting and Glimm's method to solve the system.

2. Outline of the Method

To solve the system of equations (2) using Glimm's method, we need to reduce them to conservative form without the inhomogeneous term. To handle this we proceed in the format of Sod [6], by first removing the nonhomogeneous term $-W(U)$ from Eq. (1) and solving

$$U_t+F(U)_r=0 \tag{4}$$

Glimm [3] has solved systems of the form (4) by his random choice method, which was later modified by Chorin [4] for hydrodynamics.

The results of the system (4) are then used to solve the inhomogeneous term in the system of ordinary differential equations

$$U_t=-W(U). \tag{5}$$

3. Equations of State for Water

By definition the specific internal energy e can be written in the form

$$e(S,\rho)=e_1(\rho)+e_2(S)$$

for a waterlike substance, where ρ is the density, and S is the specific entropy. The pressure p and temperature T are given by

$$p = -\partial e/\partial(1/\rho), \qquad T = \partial e/\partial S. \tag{6}$$

Under all conditions, therefore, $p = p(\rho)$. For the present analysis the Tait equation, written in the form

$$p = B[(\rho/\bar{\rho})^{\gamma} - 1], \tag{7}$$

will be used to relate the pressure and density in the water regions. Here $\gamma = 7$, $B = 3268$ atms, $\bar{\rho} = 9.233 \times 10^{-4}$ atm-sec/ft using data obtained for sea water by Richardson *et al.* [8]. The speed of sound in the liquid is given by

$$a = (dp/d\rho)^{1/2} = [\gamma(p+B)/\rho]^{1/2} = (\gamma\bar{p}/\rho)^{1/2}, \tag{8}$$

where, henceforth,

$$\bar{p} = p + B.$$

We note that the Tait equation yields the relation

$$\bar{p}/\rho^{\gamma} = \text{Const.} \tag{8a}$$

4. Riemann Problem

To solve (4) as a given initial value problem, a solution of the system is found satisfying the initial conditions $U(r, 0) =$ given function of r.

We approximate $U(r, t)$ at points $(i\,\Delta r, n\,\Delta t)$ by $\tilde{u}_i^n = U(i\,\Delta r, n\,\Delta t)$. By representing the initial distribution of $U(r, 0)$ by step functions, i.e.,

$$U(r, 0) = U(i\,\Delta r, 0) \qquad (i - \tfrac{1}{2})\,\Delta r < r \leqslant (i + \tfrac{1}{2})\,\Delta r, \qquad i = 0, 1, 2, \ldots,$$

we create a sequence of Riemann problems. We then solve each Riemann problem (system (4)) along with the constant initial data

$$\begin{aligned} U(r, n\,\Delta t) &= \tilde{u}_{i+1}^n \qquad && r > (i + \tfrac{1}{2})\,\Delta r \\ &= \tilde{u}_i^n && r \leqslant (i + \tfrac{1}{2})\,\Delta r \end{aligned} \tag{9}$$

for each adjoining cell. (For the sake of brevity, the reader is referred to Sod [6], Li and Holt [7] for a complete detailed account of Glimm's method and the handling of boundary conditions.)

System (4) is solved as a 1-dimensional shock tube problem. We thus have different given states on each side of the diaphragm for $t < 0$:

$$\begin{aligned} U(r, 0) &= S_l = (\rho_l, u_l, p_l) \qquad r < 0 \\ &= S_r = (\rho_r, u_r, p_r) \qquad r \geqslant 0. \end{aligned} \tag{10}$$

At $t = 0$, the diaphragm is ruptured, and a shock wave propagates to the left and a centered expansion wave to the right (or vice versa, depending on the initial data). Figure 1 shows, for $t > 0$, the different wave possibilities. Therefore, either a shock or expansion wave propagates out, this being denoted by S_l, S_r. S_* is the steady state region unaffected as yet by the waves, whereas S_1, S_2 denote the areas affected by the propagating waves. The lines l_1, l_2 serve to separate these two regions.

Across a shock wave in a perect gas we have the two discontinuity conditions:

$$\rho_1 u_1 v_1 - \rho_0 u_0 v_0 = p_0 - p_1, \tag{11}$$

$$\rho_1 v_1 = \rho_0 v_0 = -M, \tag{12}$$

with $v = u - \dot{\xi}$, where $\dot{\xi}$ represents the speed of the shock, subscripts 0 and 1 denote conditions on the left and right sides of the shock, respectively.

For water, we modify (11) to

$$\rho_1 u_1 v_1 - \rho_0 u_0 v_0 = \bar{p}_0 - \bar{p}_1. \tag{11a}$$

Combining Eqs. (11) and (12) for the right wave, we obtain

$$M_r = -\rho_r(u_r - \dot{\xi}_r) = -\rho_*(u_* - \dot{\xi}_r)$$

or (13)

$$M_r = \left(\frac{p_* - p_r}{u_* - u_r}\right).$$

Eliminating $\dot{\xi}_r$ from Eqs. (11), (12), we have

$$(u_r - u_*)^2 = (1/\rho_* - 1/\rho_r)(p_* - p_r)$$

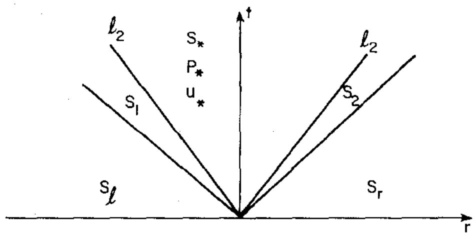

FIG. 1. The boundaries in a Riemann problem.

or

$$u_* - u_r = \left(\frac{\rho_* - \rho_r}{\rho_r \rho_*}\right)^{1/2} (p_* - p_r)^{1/2}. \tag{14}$$

Substituting Eq. (14) into Eq. (13) yields

$$M_r = \left[(p_* - p_r) \Big/ \left(\frac{\rho_* - \rho_r}{\rho_* \rho_r}\right)\right]^{1/2}. \tag{15}$$

For gas, we use the Rankine–Hugoniot equations to obtain

$$M_r = (\rho_r\, p_r)^{1/2} \left(\frac{\gamma+1}{2} x + \frac{\gamma-1}{2}\right)^{1/2}, \qquad \text{where} \quad x \geqslant 1, \quad x = p_*/p_r. \tag{16}$$

For water, we use Eq. (8a), with $\gamma = 7$, to obtain

$$\begin{aligned} M_r &= (\bar{p}_r \rho_r)^{1/2} [(\bar{p}_*/\bar{p}_r - 1)/(1 - \rho_r/\rho_*)]^{1/2} \\ &= (\bar{p}_r \rho_r)^{1/2} [(\bar{p}_*/\bar{p}_r - 1)/(1 - (\bar{p}_r/\bar{p}_*)^{1/\gamma}]^{1/2} \end{aligned}$$

or

$$M_r = (\rho_r\, \bar{p}_r)^{1/2} [(x-1)/(1-(1/x)^{1/\gamma})]^{1/2}, \qquad \text{where} \quad x \geqslant 1, \quad x = \bar{p}_*/\bar{p}_r \tag{16a}$$

If the left wave is a shock, we obtain

$$M_l = (p_l - p_*)/(u_l - u_*) \tag{17}$$

and by similar analysis to the above, we obtain the following relations:

For air

$$M_l = (\rho_l\, p_l)^{1/2} \left(\frac{\gamma+1}{2} x + \frac{\gamma-1}{2}\right)^{1/2} \qquad x = p_*/p_r \geqslant 1. \tag{18}$$

For water

$$M_l = (\rho_l \bar{p}_l)^{1/2} [(x-1)/(1-(1/x)^{1/\gamma})]^{1/2} \qquad x = \bar{p}_*/\bar{p}_r \geqslant 1. \tag{18a}$$

If the right wave is an expansion wave, the Riemann invariants, valid for both gas and water, may be used. Namely,

$$u_r - \frac{2}{\gamma-1} a_r = u_* - \frac{2}{\gamma-1} a_*$$

or

$$u_r - u_* = \frac{2}{\gamma-1} (a_r - a_*). \tag{19}$$

Substituting Eq. (19) into Eq. (13) yields, for gases,

$$M_r = (p_r\rho_r)^{1/2}\left(\frac{\gamma-1}{2\gamma^{1/2}}\frac{(1-x)}{(1-x^{(\gamma-1)/2\gamma})}\right) \qquad x = p_*/p_r < 1. \tag{20}$$

For water we use Eqs. (8), (8a) to obtain

$$M_r = (p_r\rho_r)^{1/2}\left(\frac{\gamma-1}{2\gamma^{1/2}}\frac{1-x}{1-x^{(\gamma-1)/2\gamma}}\right) \qquad x = \bar{p}_*/\bar{p}_r < 1. \tag{20a}$$

The corresponding relations for an expansion wave on the left are

For gases

$$M_l = (p_l\rho_l)^{1/2}\left(\frac{\gamma-1}{2\gamma^{1/2}}\frac{1-x}{1-x^{(\gamma-1)/2\gamma}}\right) \qquad x = p_*/p_l < 1. \tag{21}$$

For water

$$M_l = (\bar{p}_l\rho_l)^{1/2}\left(\frac{\gamma-1}{2\gamma^{1/2}}\frac{1-x}{1-x^{(\gamma-1)/2}}\right) \qquad x = \bar{p}_*/\bar{p} < 1. \tag{21a}$$

Summarizing, the expansion wave relations in gases are

$$\begin{aligned} M_l &= (p_l\rho_l)^{1/2}\,\phi(p_*/p_l), \\ M_r &= (p_r\rho_r)^{1/2}\,\phi(p_*/p_r), \end{aligned} \tag{22}$$

where

$$\begin{aligned} \phi(x)&\left[\frac{\gamma+1}{2}x+\frac{\gamma-1}{2}\right]^{1/2} && x \geqslant 1 \\ &= \left[\frac{\gamma-1}{2\gamma^{1/2}}\frac{1-x}{1-x^{(\gamma-1)/2\gamma}}\right] && x < 1. \end{aligned}$$

The corresponding relations in water are

$$\begin{aligned} M_l &= (\bar{p}_l\rho_l)^{1/2}\,\phi_1(\bar{p}_*/\bar{p}_l), \\ M_r &= (\bar{p}_r\rho_r)^{1/2}\,\phi_1(\bar{p}_*/\bar{p}_r), \end{aligned} \tag{23}$$

where

$$\begin{aligned} \phi_1(x) &= [(x-1)/(1-(1/x)^{1/\gamma})]^{1/2} && x \geqslant 1 \\ &= \left[\frac{\gamma-1}{2\gamma^{1/2}}(1-x)/(1-x^{(\gamma-1)/2\gamma})\right] && x < 1. \end{aligned}$$

When $(x-1)$ is small, it is advisable to represent $\phi(x)$ and $\phi_1(x)$ by binomial expansions.

On elimination of u_* from Eqs. (13) and (17), we obtain

$$p_* = (u_l - u_r + p_r/M_r + p_l/M_l)/(1/M_r + 1/M_l). \tag{24}$$

The Eqs. (24) and (22) (gas alone), Eqs. (24) and (23) (water alone) or Eqs. (22) and (23) (gas and water) provide three relations for three inknowns p_*, M_r, M_l. After choosing a starting value p_*^0 (or M_l^0 or M_r^0) we solve these relations by iteration. Here we choose $p_*^0 = \frac{1}{2}(p_l + p_r)$ (for details on the iteration procedure, and choice of starting value, see Chorin [4]). After p_*, M_l, M_r have been determined, we may obtain u_* by eliminating p_* from Eqs. (13) and (17). We obtain

$$u_* = (p_l - p_r + M_l u_l + M_r u_r)/(M_l + M_r). \tag{25}$$

The method of choosing the random numbers most efficiently is fully described in Chorin [4].

5. Solution of the Ordinary Differential Equations

The operator splitting technique used by Sod [6] is a predictor–corrector method applied to Glimm's method after every full time step. We follow here the modification of the method of Li and Holt [7] which essentially updates the solution at every half time step. As noted by Li and Holt, this requires no additional work, and the resolution is greatly enhanced. For full details of the scheme, see [7].

6. Application: Release of a Sphere of Gas at High Pressure

A spherical diaphragm of radius r, located at a depth h below the surface of the ocean, is filled with gas at high pressure p_{0g}. At time $t = 0$, the diaphragm bursts, sending out a spherical shock, followed by a contact surface. This contact surface separates the gaseous and water regions. Behind the contact discontinuity an expansion wave moves towards the axis $r = 0$. A weak secondary shock is formed shortly after time $t = 0$ and at first travels outwards briefly. This shock then starts to strengthen as it turns to propagate inwards, and eventually collapses on the axis. This problem has been considered by Friedman [2] and Chan *et al.* [9]. In applying Glimm's method to this problem, we check the accuracy with which the key surfaces of discontinuities are followed. The initial conditions considered are given in Table I.

In the analysis that follows we non-dimensionalize the system (1) in terms of the following constants. All lengths are divided by h, the depth at the center of the sphere $r = \tilde{r}/h$ (($\tilde{\ }$) denotes dimensional qualities). Velocities are divided by a_{0w}, the speed of sound in the undisturbed ocean, so we write $u = \tilde{u}/a_{0w}$. The density is divided by ρ_{0w},

TABLE I

Values of Basic Parameters

Initial radius of gas sphere	1/3 ft
Depth of gas sphere center	1 ft
Initial pressure of explosion gas	9000 atm
Initial explosion gas temperature	2500°K
Specific heat ratio in explosion gas	1.4

the density in the undisturbed water $\rho = \tilde{\rho}/\rho_{0w}$. The pressures are divided by $p_{0w} + B$ (= 3269 atms), $p_{0w} = 1$ atms in undisturbed water, $p = \tilde{p}/(p_{0w} + B)$ and time is divided by (h/a_{0w}), $t = \tilde{t}/(h/a_{0w})$. The space step is chosen as $\Delta r = 0.01$, while the time step is chosen to satisfy $\max(|u| + a)\,\Delta t/\Delta r \leqslant 1$, where a is local sound speed.

In Fig. 2, the pressure distribution is displayed at time intervals of 0.05 apart. Note the sharpness of the shock, the number of zones for this variation is zero. With increasing time, the shock is expanding towards the ocean surface. The shock is weakening as it propagates outwards, as indicated by the pressure rise behind it. Also, the pressure at a point behind the shock is decaying with time as would be expected (see Cole [10]).

In Fig. 3, the density distribution has the basic properties of the shock as does the pressure distribution. The density jump is not as great across the shock, as expected, since we are dealing with sea water. In the density profiles, a contact surface appears propagating outwards. It is due to Glimm's method that this contact surface remains perfectly sharp.

In Fig. 4, the velocity distribution is displayed. The particle velocity is greater near the expansion wave than right after the shock. Again, the clarity of the shock is maintained.

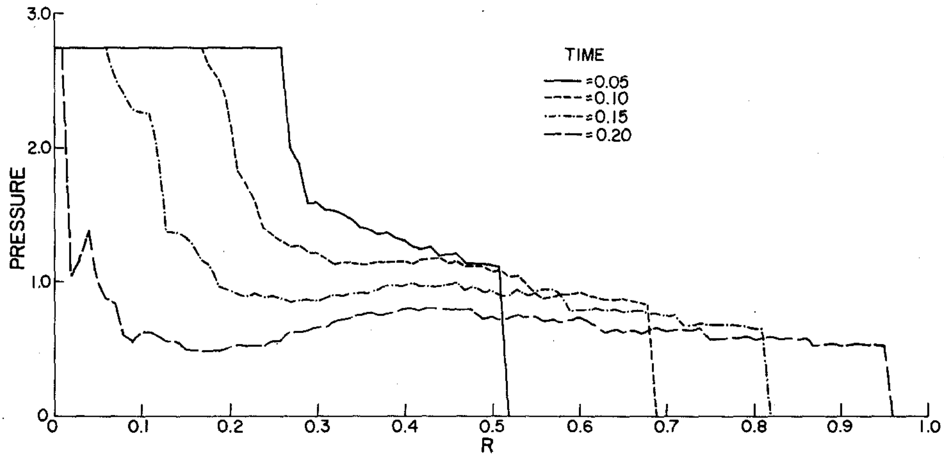

FIG. 2. The pressure distribution behind a spherical blast wave.

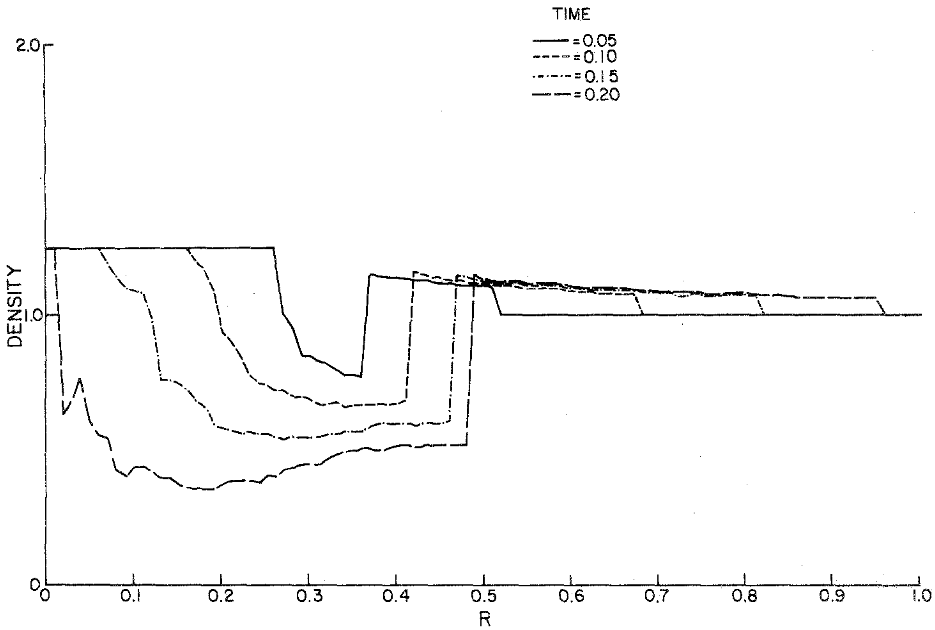

FIG. 3. The density distribution behind a blast wave.

In Fig. 5, the contact surface and shock paths are plotted. This agrees qualitatively with a similar plot by Ballhaus and Holt [1]. There were quantitative differences in the speed of the underwater shock and contact surface. This was due to the fact that in the present study air was used in the interior of the pressurized sphere, whereas helium was used in [1].

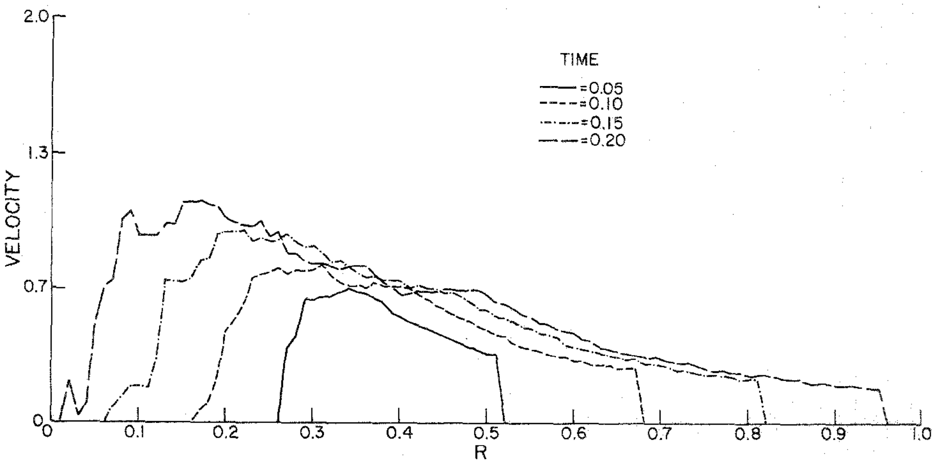

FIG. 4. The velocity distribution behind a blast wave.

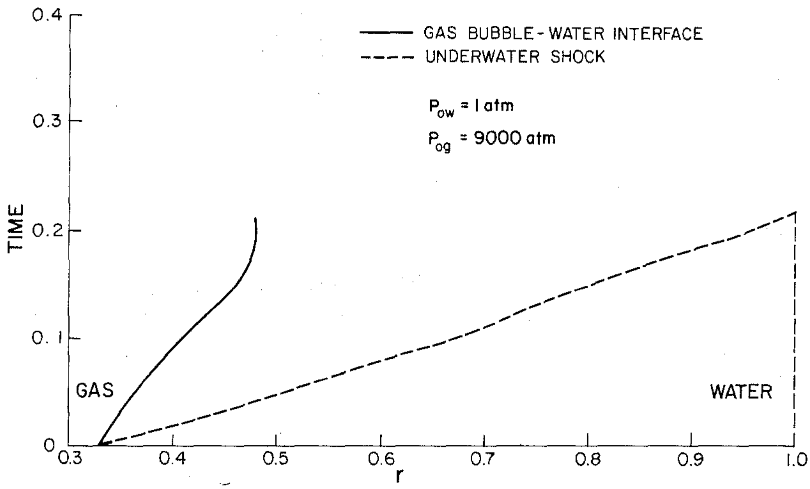

FIG. 5. Paths of underwater blast wave and gas–water interface.

7. Conclusions

In general, the qualitative results agree with what we expected to observe in this phenomenon. Glimm's method does in fact track the shock and slip line with perfect clarity, with no smearing as compared to other methods. Also, the simplicity of the method makes it a very inexpensive program to run and allows us means to explore and observe other qualitative results (and rough quantitative results) at a very low cost.

At a given time, however, thought should be given to the fact that the shock and/or contact surface may not be exact due to the randomness of the Glimm's method. Yet, on the average, their positions are exact. The roughness in the expansion wave is due to the randomness of the method.

The operator splitting appears to add error to the numerical solutions. A numerical study on this effect has been done (see [11]) and there exists a new random sampling method to help keep these errors down. The authors have a copy available of the new method.

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